Central European University
Department of Economics

1. **Name of Course**: Stochastic Methods and Mathematical Finance

2. **Lecturer: Professor Péter Medvegyev**

3. **No. of Credits (no. of ECTS credits): 2 CEU credits (4 ECST)**

4. **Semester or Time Period of the course:** Winter term

5. **Pre-requisites:** No pre-requisites, Advanced Mathematics is suggested

6. **Course level:** Elective course for master or for PhD. students

7. **Course Outline:** The course gives an introduction to the theory of stochastic processes and its applications to mathematical finance.

8. **The goals of the course:** To give some solid introduction to the fundamental concepts of the theory of stochastic processes and to the application of the theory to mathematical finance.

9. **The learning outcomes of the course:** The students will be able to understand some concepts of the theory of stochastic processes and mathematical finance.

10. **More detailed display of contents**:

Assumed Background:

Interest in mathematics and understanding of basic concepts of calculus, probability theory.

Preliminary Syllabus:

Stochastic processes in discrete and continuous time
Different type of stochastic processes, Wiener, Poisson and Markov processes
Poisson processes as counting Lévy processes, the role of the exponential and gamma distributions
Credit Risk+ model
Wiener processes and the central limit theorem
Martingals, local martingals and semimartingals
Stochastic integration and Ito's formula
Basic concepts of mathematical finance

Option pricing theory, Black-Scholes formula
Different type of options and pricing of derivatives.

11. **Assessment:** Final examination

Preliminary Weekly Breakdown

Lecture 1

- Continuous and discrete time stochastic processes
- Filtration, stopping times
- Interpretation of the mathematical concepts of the theory
- Construction of stopping times.

REFERENCES

Medvegyev: Handout

Lecture 2

- Processes with independent increments, Lévy processes
- Poisson processes as counting Lévy processes
- Jump times of Poisson processes
- The time of the first jump has exponential distribution
- The number of the jumps has Poisson distribution

REFERENCES

Medvegyev: Handouts

Lecture 3

- Problem of credit risk, Value at Risk
- Definition of the credit and operational risk
- The credit risk+ model
- Calculation a default probabilities

REFERENCES

Medvegyev: Handout
Credit risk+ documentation

Lecture 4

- Conditional expectation
- Properties of the conditional expectations
- Markov processes and martingales
- Optional Sampling Theorem

REFERENCES

Medvegyev: Handout

Lecture 5

- Stochastic integrals
- Interpretation of stochastic integrals
- Why stochastic integrals are not martingales
- Local martingales

REFERENCES

Medvegyev: Handout
Thomas Björg: Arbitrage Theory in Continuous Time, Oxford University Press, 1998, Chapter 3

Lecture 6

- Wiener processes
- Quadratic variation of Wiener processes
- Quadratic variation of local martingales
- Pricing of options and general formula for pricing derivative products: compensators and semimartingales

REFERENCES

Medvegyev: Handout
Thomas Björg: Arbitrage Theory in Continuous Time, Oxford University Press, 1998, Chapter 3

Lecture 7-8

- Transformed random variables and their distribution
- Transformed stochastic processes
- Ito’s formula
- Application of Ito’s formula
- Characterization of martingales, when a local martingale is a real martingale
- Expected value of transformed processes

REFERENCES

Medvegyev: Handout
Thomas Björg: Arbitrage Theory in Continuous Time, Oxford University Press, 1998, Chapter 3

Lecture 9

- Stock returns and stock prices
- The Black-Scholes model of dynamics of the prices of the stocks
- The lognormal distribution
- Main properties of the distribution of the stock prices

REFERENCES

Medvegyev: Handout
Thomas Björg: Arbitrage Theory in Continuous Time, Oxford University Press, 1998, Chapter 5

Lectures 9

- The Black-Scholes differential equation
- The Black-Scholes differential equation and the option pricing theory
- Risk neutral measures and pricing of options

REFERENCES

Medvegyev: Handout
Thomas Björg: Arbitrage Theory in Continuous Time, Oxford University Press, 1998, Chapter 4.

Lectures 11-12

- Change of measure
- Girsanov's theorem
- The integral representation theorem, the completeness of the market
- The Black-Sholes formula
- Application of Black-Scholes formula

REFERENCES

Medvegyev: Handout
Thomas Björg: Arbitrage Theory in Continuous Time, Oxford University Press, 1998, Chapter 6 and Chapter 7