Assume that X is continuous in probability and X has independent increments. By Corollary 7.92 the Fourier transform of X is

$$\begin{aligned}\varphi(u,t) &= \exp(\Psi(u,t)) = \\ &= \exp(iuB(t) - \frac{1}{2}uC(t)u + (L(u,x) \bullet \nu(x))(t)).\end{aligned}$$

As the jumps are bounded and as ν is deterministic one can write the integral part as

$$\begin{aligned}(L(u,x) \bullet \nu(x))(t) &= \int_{(0,t]\times K} \exp(iux) - 1 - iuxd\nu(x) = \\ &= \int_K \exp(iux) - 1 - iuxd\nu_t(x)\end{aligned}$$

where K is a bounded set containing the jumps of X. It is well-known that the even order derivatives of the Fourier transform of a random variable are finite if and only if the same order of moments are also finite. That is

$$\mathbf{E}\left(|X(t)|^{2m}\right) < \infty$$

if and only if

$$\varphi^{(2m)}(0,t) \quad \text{exists and it is finite.}$$

Obviously

$$\begin{aligned}\int_K \exp(iux) - 1 - iuxd\nu_t(x) &\overset{\circ}{=} \int_K x^2 h(x,u)\, d\nu_t(x) \overset{\circ}{=} \\ &\overset{\circ}{=} \int_K h(x,u)\, d\mu(x)\end{aligned}$$

where

$$\mu(K) = \int_K x^2 d\nu_t(x) < \infty.$$

It is easy to see that the integrand h and all of its derivatives with respect to u are bounded in some neighborhood of $u = 0$ and as $\mu(K) < \infty$ one can easily differentiate under the integral sign. Hence all the even moments, therefore all the moments of $X(t)$ are finite.